APPENDIX - C

THE ILLINOIS COORDINATE SYSTEM

BUREAU OF DESIGN AND ENVIRONMENT

SURVEY MANUAL

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Appendix - C

THE ILLINOIS COORDINATE SYSTEM

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DESCRIPTION OF THE ILLINOIS PLANE COORDINATE SYSTEM

I. INTRODUCTION

The National Geodetic Survey uses geodetic surveying procedures to compute all of its surveys. The results obtained from computing geodetic data are precise but require the use of complex spherical formulas and special tables. The complexity of these computations and the demand for being able to make use of this vast amount of information prompted the Federal Board of Surveys and Maps to recommend that its member organizations adopt the plane rectangular coordinate systems. Plane coordinate data is not exact since allowances must be made in the transfer of the survey data from the surface of the Earth (which is curved in latitude and longitude) to the plane surface of the grid system which is curved in only one dimension. The computations are made with the ordinary formulas of trigonometry.

Two projections were selected for use in plane coordinate systems. The Lambert Projection was selected for use in States whose longitudinal axis is East-West and the Transverse Mercator Projection was selected for States whose longitudinal axis is North-South. The Illinois State Plane Coordinate System is based on the Transverse Mercator Projection.

The North American Datum of 1927 (NAD27) uses Clarke's spheroid of 1866 for its reference ellipsoid, which is a best fit for the United States. The NAD83 uses the Geodetic Reference System (GRS80), an ellipsoid which best fits the overall shape of the entire Earth. The published values of latitude and longitude of the National Geodetic Survey horizontal control positions are referenced to these mathematical figures. The NAD83 values have replaced the NAD27.

In 1997 and 1998 new surveys were done in Illinois using the Global Positioning System to resurvey the state to a higher degree of accuracy. All National Geodetic Survey horizontal control monuments in Illinois reflect the new adjustment values. The latest adjusted values should always be used for new projects unless directed not to do so by the project engineer.

One of the primary uses of a State Plane Coordinate System is to make possible the correlation of surveys in all areas of the system. The positions of the grid lines of a state coordinate system are determined with respect to the meridians and parallels on the spheroid (ellipsoid) of reference. Therefore, a point that is defined by stating its latitude and longitude can also be defined by stating its X and Y coordinates on a grid. If either set of values is known, the other set can be computed. This is true also with lengths and azimuths.

The geodetic length and azimuth between two positions can be transformed into a grid length and azimuth or the inverse by mathematical computations. A computer program is available for determining the geodetic position from plane coordinates and vice versa.

II. CONVERGENCE OF MERIDIANS

The curvature of the Earth causes meridians of longitude to converge. Any two meridians intersect at the North Pole with an angular value equivalent to the difference between the values of the two meridians. This characteristic of convergence is demonstrated in Figure C.1, page C-6 showing lines tangent to 88 degrees and 89 degrees longitude at the equator and again at the Pole.

The effect of this convergence, (θ), on the survey line is found by multiplying the difference in the longitude ($\Delta \lambda$) of the ends of the line by the sine of the mean latitude (Φ) of the line. For example: Calculate the convergence of the meridians between two points whose geographic coordinates are:

Latitude	45° 15' 15"	45° 10' 15"
Longitude	75° 13' 28"	75° 10' 12"
The mean latitude is	45° 12' 45"	

The difference in longitude is 3' 16" = 196"

$\theta = \Delta \lambda \sin \Phi$

$\theta = (196)\,(.70972) = 139'' = 0°\ 2'\ 19''$

In geodetic surveying this convergence must be taken into consideration because of its effect on the azimuth of traverse and triangulation lines. The forward geodetic azimuth differs in numerical value from the back azimuth by a quantity other than 180°. This quantity is expressed by the equation $a = a' + 180 + \Delta \theta$; where a is the forward azimuth, a' is the back azimuth and $\Delta \theta$ is the numerical difference. This can be seen in Figure C.2, page C-6. Meridians are drawn through points A and B which in each case represents true North. Angles "a" and "b" are not equal because of the convergence, therefore, azimuth values at A & B are not equal by that amount.

III. THE COORDINATE SYSTEM

The Illinois coordinate system is made up of two zones, each having a central meridian. A central meridian is located at 88° 20' West Longitude for the East Zone and at 90° 10' West

Longitude for the West Zone. The latitude of the origin of the zones is 36° 40'. These zones were created by making a cylinder secant to the spheroid (ellipsoid). The zones overlap a small amount to facilitate the transfer of data from one to the other. When the system was devised, the scale along the central meridian of the East Zone was made 1 part in 40,000 to small. Therefore, 28 miles East or West of this central meridian the scale is exact. The scale beyond this distance is too large. The scale along the central meridian of the West Zone was made 1 part in 17,000 too small. Therefore, in this case the point of exact scale is reached at 43 miles East or West of the central meridian. Figure C.3, page C-7 shows graphically the relationship between the plane coordinate system and the lines of latitude and longitude, where λ is the longitude of a point and Φ is the latitude of a point.

The X coordinate of each central meridian is assigned the value of 500,000 feet (NAD27). For the NAD83 the central meridians are assigned different values. The East Zone is 300,000 meters and the West Zone is 700,000 meters. The distance from the central meridian to any point East or West of it is called X'. X' = X-500,000 or X' = 500,000 – X. This equation can be used for both the NAD27 and the NAD83 values to determine the scale factor. The projection parameters for Illinois did not change when the NAD83 values were established.

At the central meridian, grid North is the same as geodetic North. At all other points in the system, there is an angular difference between them. This angular difference is represented by $\Delta\ \alpha$. Its value is the product of the sine function of the latitude of the point and the seconds of longitude between the central meridian and the point ($\Delta\ \alpha = \Delta\ \lambda''$ sine Φ).

Figure C.4, page C-7 shows that West of the central meridian, grid North is West of true North and grid azimuths are numerically greater than geodetic azimuths. For positions East of the central meridian, the reverse is true. This representation also clearly points out how the delta alpha angle should be applied to geodetic azimuths to obtain plane azimuths. It is apparent that the difference between grid azimuth values and geodetic azimuth values increases as work progresses East or West from the Central meridian. In Illinois, the delta-alpha angle ($\Delta\ \alpha$) amounts to approximately one minute of arc per mile.

The National Geodetic Survey has prepared two publications that are useful in computing coordinates on the Illinois Plane Coordinate System. Special Publication No. 303 “Plane Coordinates Projection Tables” illustrates a method and gives values to be used in transforming geodetic positions to plane coordinates and the inverse. Special Publication No. 235 “The State Coordinate System” is a manual designed to aid the surveyor in using the system.

IV. REDUCTION OF GROUND LENGTHS

A survey to be placed on the State Plane Coordinate System must have a sea level (elevation) correction and a scale correction applied to the ground measured distances.

The elevation factor to be applied to convert the ground distances to sea level distances is computed by the following equations. The elevation factor = R/(R+h) where h is the elevation of the ground above sea level and R is the mean radius of curvature of the sea level spheroid across Illinois, which is 20,906,000 feet. This elevation factor is used for the NAD27. For the NAD83, the elevation factor is = R/(R+N+H). where N is the geoid-ellipsoid separation, and H is the elevation above sea level. In Illinois the value of N is always negative. It should be applied algebraically in the formula. The average elevation to be used can be obtained from U. S. G. S. Quadrangles.

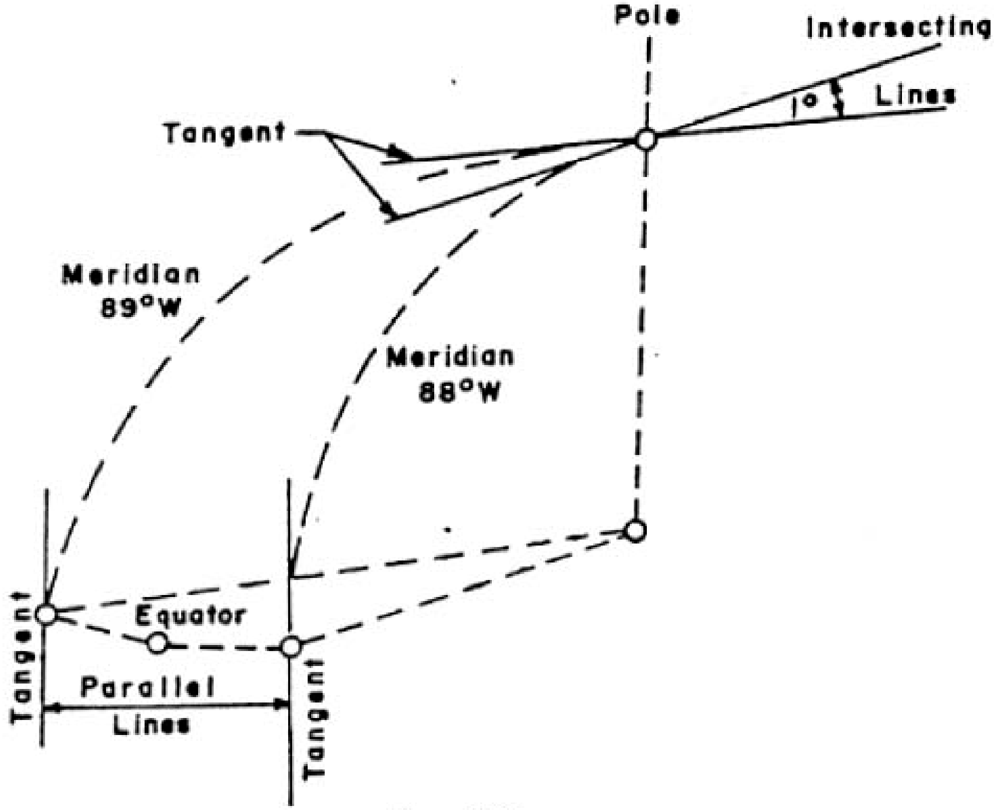
The factor to be applied to reduce the ground distance to the plane of the cylinder is found in the tables on pages 27, 28 and 29 in Special Publication No. 303. The X' value shown on these pages is that described above . The scale factor may be expressed as a ratio of grid length to geodetic length or it may be given as a numerical correction to be applied to the measured length reduced to sea level. The scale difference and therefore the scale factor decreases with the distance from the central meridian. When traverse or triangulation lines exceed five miles in length, it becomes necessary to investigate the contributing elements and use judgement in applying the proper computations to establish a scale factor. There is no set formula that can be applied. For example, a traverse line running North-South near 352,160 E (East Zone) and 647,840 E (East Zone) would have no scale correction. The scale factor becomes unity near these grid lines. For survey lines that run predominantly East-West for some distance from 352,160 E and 647,840 E, the scale factor changes rapidly. For surveys with lines in excess of five miles in length and expected to have an accuracy of 1 part in 10,000 or better, the scale factor must be an average of that at each end and at the center of the survey. The equation:

$$\frac{1}{K} = \frac{1}{6}\left(1/K_1 + 4/K_3 + 1/K_2\right)$$

produces this refinement. The terms K1 and K2 represents the scale factor for the end points of the line and K3 the midpoint. In cases where elevation factors and scale factors both must be used, it is advisable to combine these factors for simplicity. This combined factor is normally referred to as the “grid factor”.

V. PLANE AZIMUTHS AND COORDINATES

The azimuth of a line is its direction and is expressed as the clockwise angle between grid North and the line. Grid north is assigned a value of zero degrees. The advantage in using azimuths in place of bearings is that the observed angle between two lines can be applied directly to the azimuth of one line to obtain the azimuth of the other. In plane surveying, the azimuth and back azimuth of any line differ by 180°.



FigureC.1

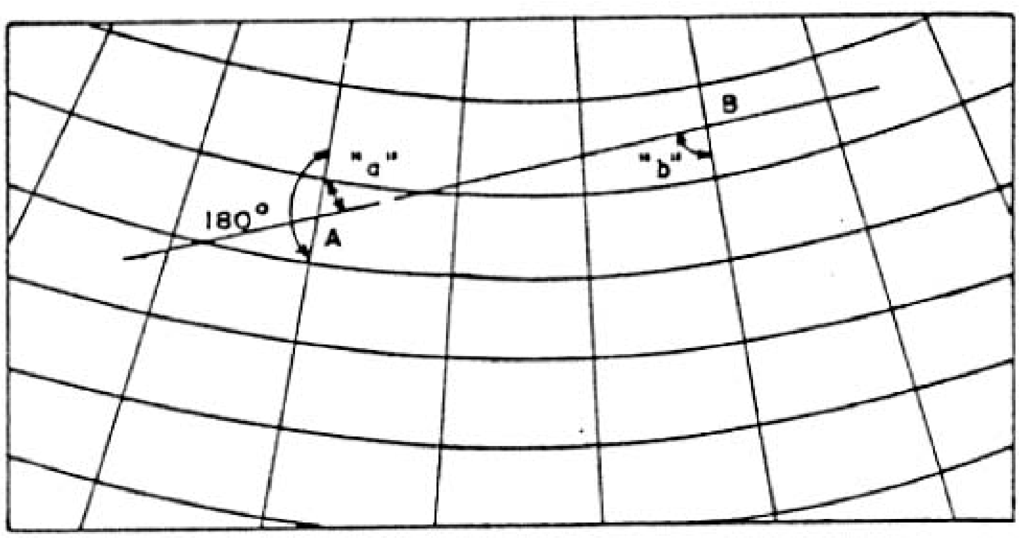


Figure C.2

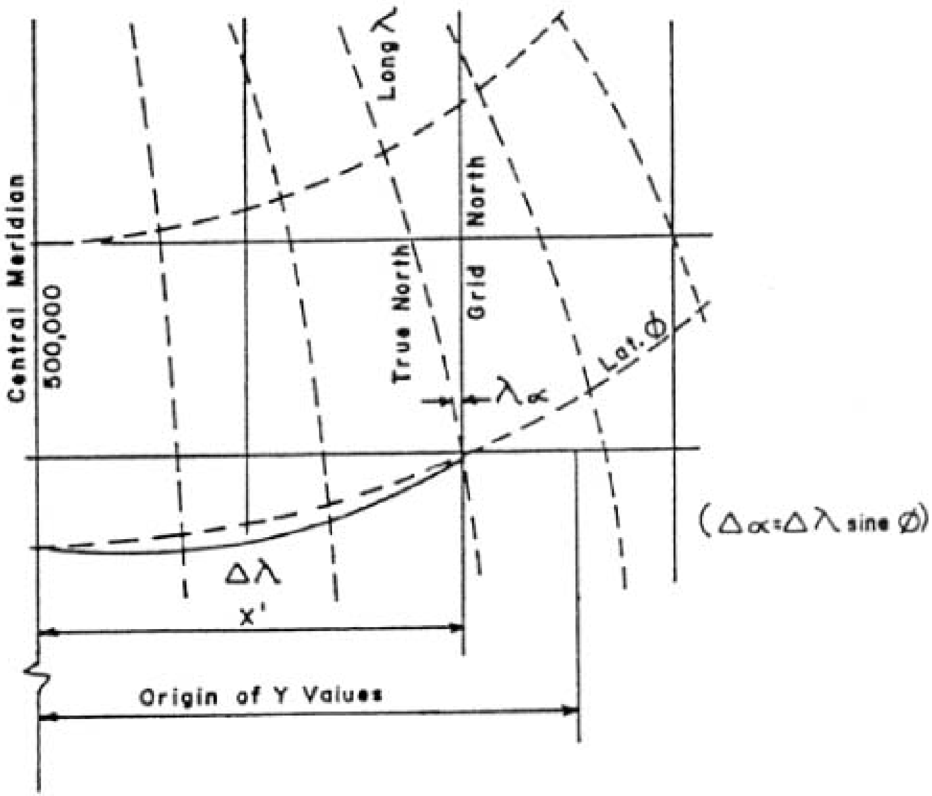


Figure C.3

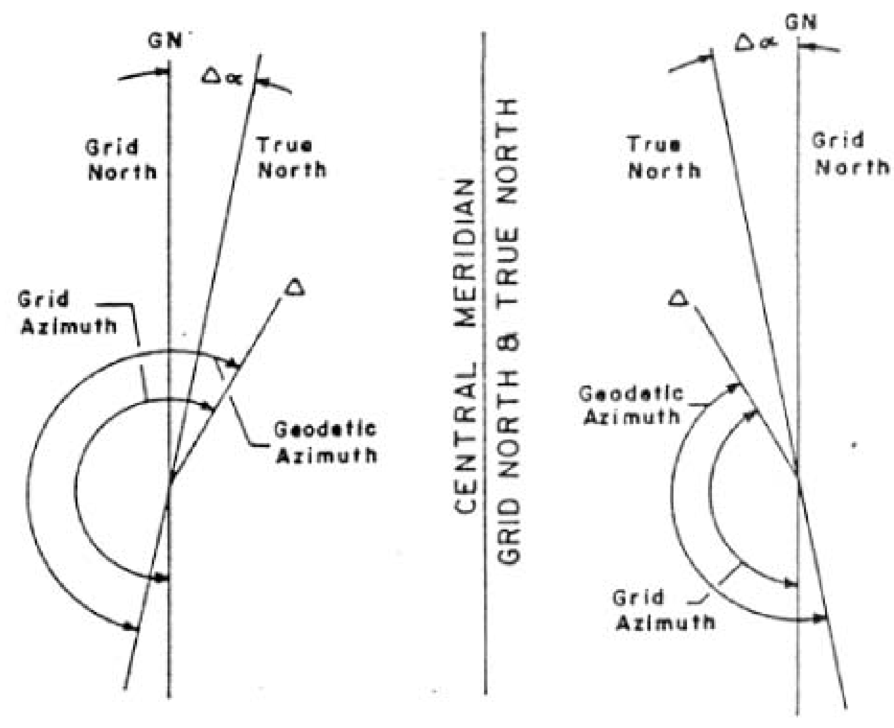


Figure C.4

VI. USING THE ILLINOIS STATE PLANE COORDINATE SYSTEM

The Illinois State Plane Coordinate System (ISPCS) is not something that is new to surveying. The system has been in existence since 1933. It has become more common now because of the Global Positioning Systems (GPS) being used for acquiring horizontal and vertical positioning. You do not have to know anything about GPS to understand a state plane coordinate system. The GPS software provides state plane coordinates as part of the output from the observed data.

The system was developed to allow a surveyor to use a plane rectangular coordinate system while surveying on a curved surface. Prior to the development of the State Plane Coordinate System, geodetic positions given in latitudes and longitudes represented the only statewide coordinate system available to surveyors. To use such a geodetic system was very impractical. It required knowledge of the tedious and complicated procedure of geodetic surveying. Geodetic surveying involves performing computations on the surface of a spheroid. The development of the SPCS has made the very precise geodetic system available to local surveyors who can now use plane-rectangular survey methods to reference their surveys to the statewide system. The system provides the means to relate geodetic positions (latitude and longitude), which are referenced to an ellipsoid, to a plane grid system which is much easier to work with.

The SPCS was developed in 1933 by the U. S. Coast & Geodetic Survey, now known as the National Geodetic Survey (NGS). A system was designed for each state of the union. Converting geodetic coordinates to state plane coordinates involves projecting survey measurements made on a curved surface onto a plane surface. In the United States two map projections are used. One is a cylinder and one is a cone. These projections allow the scale factor to be variable in only one direction, either east-west or north-south. This makes the system easier to use. The scale factor will be discussed in more detail later.

VII. PROJECTIONS:

The projections used to transfer coordinates from geodetic to state plane are the Transverse Mercator and the Lambert Conformal.

A. TRANSVERSE MERCATOR

In the Transverse Mercator Projection a cylinder rotated 90º to the axis of the Earth is used as a projection surface. As Figure 2.1, page 2-29 of Chapter Two illustrates, a cylinder having a slightly smaller diameter than the ellipsoid is intersected with the ellipsoid. An ellipsoid is a mathematical figure that best represents the shape of the Earth. Measurements made on the Earth's surface are projected onto the cylinder's surface.

The Transverse Mercator projection is used for states that have long distances in a north-south direction. The north-south lines where the cylinder and the ellipsoid intersect are known as lines of equal scale. They are designed to be equidistant from the central meridian of the zone. Along the lines of equal scale, measurements on the ellipsoid and cylinder of projection are equal.

Illinois uses the Transverse Mercator Projection. To maintain a certain degree of accuracy, two different projections are used to cover Illinois. There is the East Zone and the West Zone. To avoid confusion, coordinates used or developed in a local survey must identify which zone was used. See Figure C.5, page C-16.

A.1 East Zone

The East Zone was designed so that the Central Meridian of the zone lies along longitude 88º 20' W. The origin of the zone is at the intersection of latitude 36º 40' N and the central meridian. At the origin the North coordinate is assigned a value of zero. The central meridian is assigned a value so there will never be any negative coordinates in the east-west direction for that zone. The accuracy of the East Zone is 1:40,000 at the Central Meridian. This is based upon the amount of linear distortion that is mathematically imposed upon ellipsoid distances when projected onto the plane surface at the central meridian of the zone.

A.2 West Zone

The West Zone was designed so the Central Meridian of the zone lies along longitude 90º 10' W. The origin of the zone is at the intersection of latitude 36º 40' N and the central meridian. At the origin the North coordinate is assigned a value of zero. The central meridian is assigned a value so there will never be any negative coordinates in the east-west direction for that zone. The accuracy of the West Zone is 1:17,000 at the Central Meridian.

Figure C.6, page C-17 shows the zone constants for the East and West Zones in Illinois.

B. LAMBERT CONFORMAL

The Lambert Conformal Projection is used for states with the longest distance being in an east-west direction. Here the projection surface is a cone. The lines of equal scale are lines of latitude. See Figure 2.2, page 2-30 of Chapter Two. The scale factor does not change in an east-west direction, only in the north-south direction. Figure C.7, page C-18 illustrates the type of projection used in each state.

VIII. DISTANCE REDUCTIONS

To establish coordinates on the state plane coordinate system, two corrections must be applied to the distance measurements. Figure C.8, page C-19 provides an overview of reducing a distance from geodetic to grid or the plane surface. These corrections transfer the measurement from the ground surface to the plane surface. One correction is provided by the ***elevation factor*** which transfers the measured distance from the Earth's surface to the ellipsoid surface. The second correction is provided by the ***scale factor*** which transfers the measured distance from the ellipsoid to the plane surface.

A. ELEVATION FACTOR FOR THE NORTH AMERICAN DATUM OF 1927 (NAD27)

The elevation factor is the ratio of the distance at mean sea level to the horizontal distance on the Earth’s surface. See Figure C.9, page C-20. Using similar triangles the ratio is that of the Earth’s radius to the Earth’s radius plus the height above mean sea level. For the determination of the elevation factor the Earth’s radius is assigned a value of 20,906,000 feet. The exactness of this value is not critical when using a value of this size in a ratio. In the NAD27 the ellipsoid of reference was considered to be the same as mean sea level. Mean sea level is also referred to as the geoid. They are not exactly the same but for lower order surveys they are close enough to be considered the same. The Elevation Factor for use in computing NAD27 coordinates can be calculated by using the following equation:

E. F. = R/(R+h)

where h = Height above mean sea level

R = 20,906,000 feet

Example computation: Known Information

h = 682.35 feet

R = 20,906,000 feet

E. F. = 20,906,000/(20,906,000+682.35) = 0.99996736

B. ELEVATION FACTOR FOR THE NORTH AMERICAN DATUM OF 1983 (NAD83)

In 1983 the National Geodetic Survey (NGS) performed a new adjustment of the horizontal control network in the United States. The elevation factor for NAD83 is calculated differently than for NAD27. The ellipsoid of reference has been redefined. In the NAD27 the Clarke's spheroid of 1866 was designed to best represent the Earth's shape across the coterminous United States. In the NAD83 the ellipsoid was redefined to best fit the entire Earth. The ellipsoid of reference for NAD83 is the Geodetic Reference System of 1980 (GRS80). Figure C.10, page C-21 gives the formula for computing the elevation factor when using the NAD83 coordinate system. In this formula the separation (N) between the ellipsoid and mean sea level (geoid) must be accounted for. The value of N is given on the description sheets for stations having NAD83 coordinates. The value of N is always a negative value in Illinois because mean sea level is below the ellipsoid of reference as defined by GRS80. The Elevation Factor for NAD83 can be calculated by using the following equation:

E. F. = R/(R+N+H)

where N = Geoid-Ellipsoid Separation
H = Elevation
R = 20,906,000 feet

Example computations: Known information
H = 682.35 feet,
N = -105.754 feet.

E. F. = 20,906,000/(20,906,000-105.754+682.35) = 0.99997242

Normally, only one elevation factor is determined for a project. Determine an average elevation of the terrain and use it to determine the elevation factor.

C. SCALE FACTOR

The scale factor determination is the same for either the NAD27 or NAD83. That is because the definitions of the zones were not changed during the development of NAD83. The projections are the same. The equations to calculate the scale factor are not as simple as for the elevation factor. The NGS furnishes tables for determining the scale factor for a point or project. The interpolation is relatively easy to make to determine the scale factor for a particular project. See Figures C.11 and C.12 on pages

C-22 and C-23 for the scale factor tables for the East and West Zones in Illinois. The scale factor is based on the distance (x') that the point lies east or west of the central meridian of the zone. To obtain this distance (x') calculate the difference between the east coordinate of the point and the east coordinate of the central meridian. The value of the central meridian is different in the NAD27 and NAD83 systems. For NAD27 the central meridian has a value of 500,000 feet for both zones. In NAD83 the value for the East Zone is 300,000 meters and for the West Zone it is set at 700,000 meters.

Following is an example computation of a scale factor for a point in the East Zone. The West Zone is calculated using the tables for the West Zone.

Known Information: The point has an east coordinate (X) of = 623,234.980 feet

X' = X - 500,000

X' = 623,234.98 - 500,000 = 123,234.98 feet.

From the table for the East Zone the X' distance lies between an X' value of 120,000 and 125,000. From the table the S. F. for 120,000 is 0.9999915 and for 125,000 it is 0.9999929. It is a matter of a simple interpolation to determine that the S. F. for 123,234.98 is = 0.9999924. Normally, only one scale factor is determined for a project. If a survey project is long (10 miles or more) in an east-west direction, the project should be divided up into smaller segments and a scale factor determined for each segment. The number of segments will depend upon the accuracy desired of the survey.

D. GRID FACTOR

Sometimes you will come across a value labeled ***grid factor***. This is not another separate factor that must be determined. It is a combination of the elevation factor and the scale factor. For ease of computations the two factors are normally combined into one factor because both the elevation factor and the scale factor are applied to the distance measurements made on the surface of the Earth to convert them to a grid distance in the SPCS. The grid factor is computed by simply multiplying the two factors together.

IX. HORIZONTAL ANGLES IN THE STATE PLANE COORDINATE SYSTEM

A. HORIZONTAL ANGLE CORRECTION

So far we have been discussing how to convert the measured ground distances to grid distances for use in computing state plane coordinates. Because we are surveying on a curved surface, the horizontal angle measurements are somewhat distorted. The angles measured are not true horizontal angles because of the Earth's curvature. Corrections can be determined and applied to convert the measured angles to grid angles. But since the correction is so small, we do not normally calculate this correction unless we are performing first order or better surveys. Lines of site must be over 5 miles in length before we even start to see the effects of this distortion due to the Earth's curvature. Therefore, the corrections to the horizontal angles will not be covered here.

B. CONVERGENCE ANGLE

The convergence angle is something that is important and needs to be addressed. In the geodetic coordinate system the lines of longitude all converge at the poles. Geodetic azimuths are referenced to these lines. In a rectangular coordinate system, like the Illinois State Plane Coordinate System, all north-south lines are parallel. Grid azimuths are referenced to these parallel lines. The central meridian is the only place where there is not a convergence angle. Here the grid north is the same direction as the line of longitude. Everywhere else there is a difference between the geodetic azimuth and the grid azimuth. This difference is called the **convergence angle** or **mapping angle**. The convergence angle can be calculated for a point by using the following equation:

$\alpha'' = (\Delta\gamma'') \text{ sine } \phi^{\circ}.$

where

α'' = the mapping angle convergence expressed in seconds of arc

$\Delta\gamma''$ = the longitudinal difference, expressed in seconds of arc, between the central meridian and the point. Subtract the longitude of the point from the longitude of the central meridian.

ϕ° = the latitude of the point expressed in decimal degrees

Example computation for a Convergence Angle

A point in the East Zone has a position of 88º 23' 43" W,
39º 44' 18" N

Using the equation above,

$\Delta\gamma''$ = 88º 20' 00" minus 88º 23' 43" or -223"
sine ϕ = sine of 39º 44' 18" = 0.63928244

α'' = -223 * 0.63928244 = -142.56" or -0º 02' 22.56"

After calculating the convergence angle, the Grid Azimuth can be determined by the following equation:

Grid Azimuth = Geodetic Azimuth - α

Please Note: For azimuths that lie east of the central meridian, the grid azimuth is less than the geodetic azimuth. The opposite is true for azimuths that are west of the central meridian. The algebraic sign must be applied in the above equation. See Figure C.13, page C-24.

X. ORTHOMETRIC AND ELLIPSOIDAL HEIGHTS

There are two reference datums for measuring heights. One is the orthometric height and the other is ellipsoidal heights. Orthometric heights are referenced to the geoid while ellipsoidal heights are referenced to the ellipsoid defined by the GRS 80.

A. ORTHOMETRIC HEIGHTS

These are heights that we are all familiar with. When we perform leveling on the surface of the Earth and reference our level circuits to elevations of known bench marks, we are determining orthometric heights for the new points. These heights are referenced to the ***geoid*** or mean sea level. The ***geoid*** is defined as "the figure of the Earth considered as a sea level surface extended continuously through the continents. It is a theoretically continuous surface that is perpendicular at every point to the direction of gravity (the plumb line). It is the surface of reference for astronomical observations and for geodetic leveling." (Definitions of Surveying and Associated Terms, ACSM 1972).

B. ELLIPSOIDAL HEIGHTS

These heights are referenced to the ellipsoid defined by the Geodetic Reference System of 1980 and not mean sea level or the geoid. Ellipsoidal heights are a direct output of the GPS because GPS uses the GRS 80 as its ellipsoid of reference.

C. GEOID-ELLIPSOID SEPARATION

The Clarke's Spheroid of 1866 and the Geodetic Reference System of 1980 are two different mathematical figures. As mentioned earlier the Clarke's spheroid of 1866 provided a best fit ellipsoid for the coterminous United States, whereas the GRS80 is a best fit for the Earth. The difference between the two is called the geoid-ellipsoid separation. See Figure C.14, page C-25 for an illustration. In Illinois the ellipsoid is above the geoid. This separation must be taken into account when working with NAD83 coordinates.

XI. COMPUTING SURVEY TRAVERSES USING STATE PLANE COORDINATES

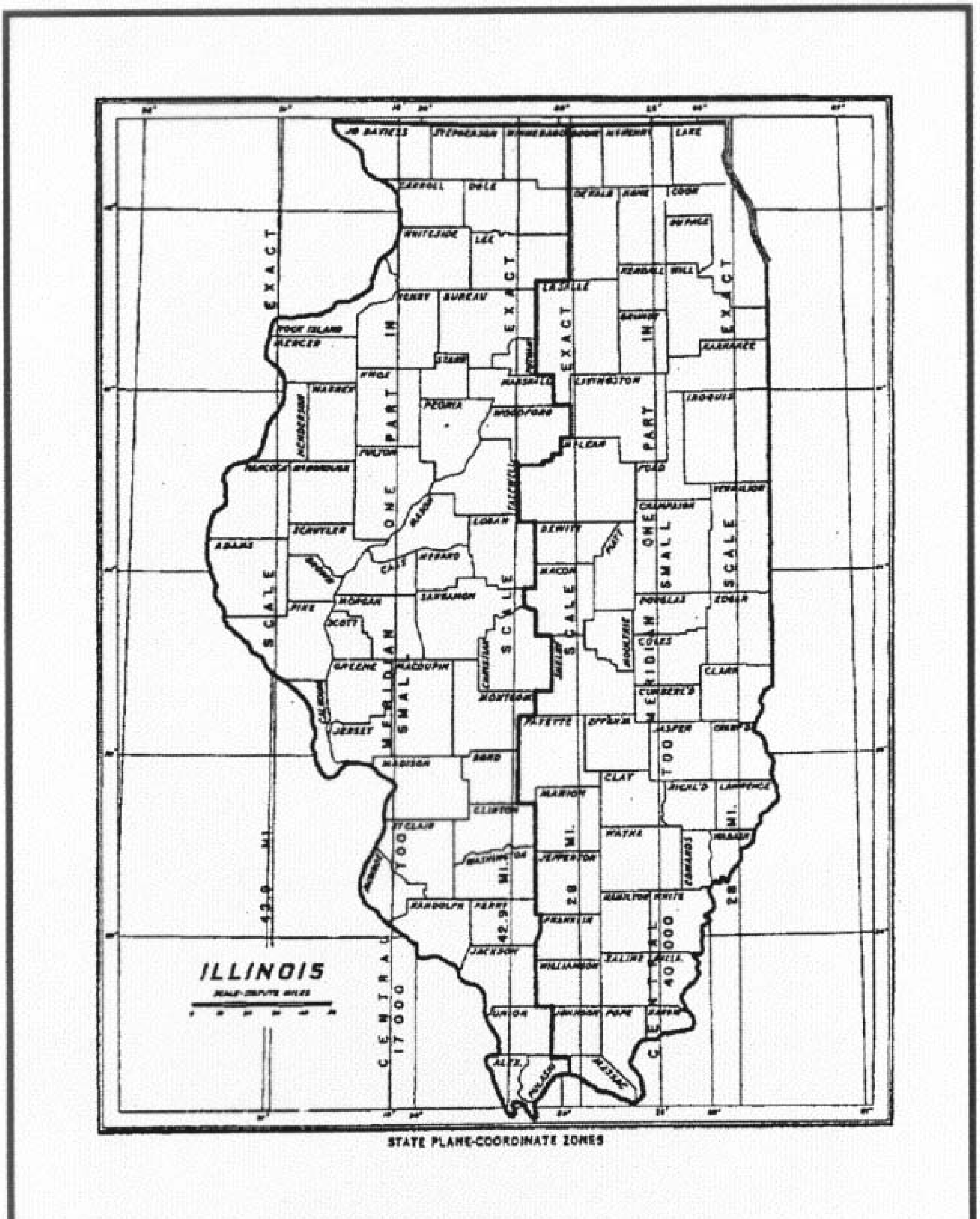
Computing a traverse to obtain state plane coordinates is basically the same as any other traverse computations. The azimuths of the courses must be calculated. Once the azimuths and distances are determined, then the latitudes and departures can be calculated just as they are in a traverse where state plane coordinates are not used.

A. CALCULATING GRID AZIMUTH

To determine the grid azimuths of the courses of a traverse, a known azimuth must be observed and a horizontal angle measured from it to the first course of the traverse. Successive angles are then added to the previous azimuth to determine the azimuth for every course in the traverse. At the last point of the traverse the horizontal angle must close onto a known azimuth to check the accuracy of the angle measurements. If the required accuracy is met for the survey, the angles are adjusted so the summation of the angles provides a perfect closure.

B. CALCULATING THE TRAVERSE

After calculating the grid azimuths and the grid distances, the latitudes and departures can be determined for each course. This part of the traverse computation follows the same procedures as for any other traverse. To obtain state plane coordinates for the traverse the beginning point of the traverse must have known coordinates from a previous survey.



STATE PLANE-COORDINATE ZONES

Figure C.5

Illinois State Plane Coordinate System Projection Constants

		NAD 27	NAD 83
East Zone (1201)			
Central Meridan	=	88° 20'	88° 20'
Origin Longitude	=	88° 20'	88° 20'
Origin Latitude	=	36° 40'	36° 40'
Origin Easting	=	500,000 ft	300,000 m (984,250 ft)
Origin Northing	=	0 ft	0 ft
West Zone (1202)			
Central Meridan	=	90° 10'	90° 10'
Origin Longitude	=	90° 10'	90° 10'
Origin Latitude	=	36° 40'	36° 40'
Origin Easting	=	500,000 ft	700,000 m (2,296,583 ft)
Origin Northing	=	0 ft	0 ft

Figure C.6

ZONES OF THE NAD83 STATE PLANE COORDINATE SYSTEM

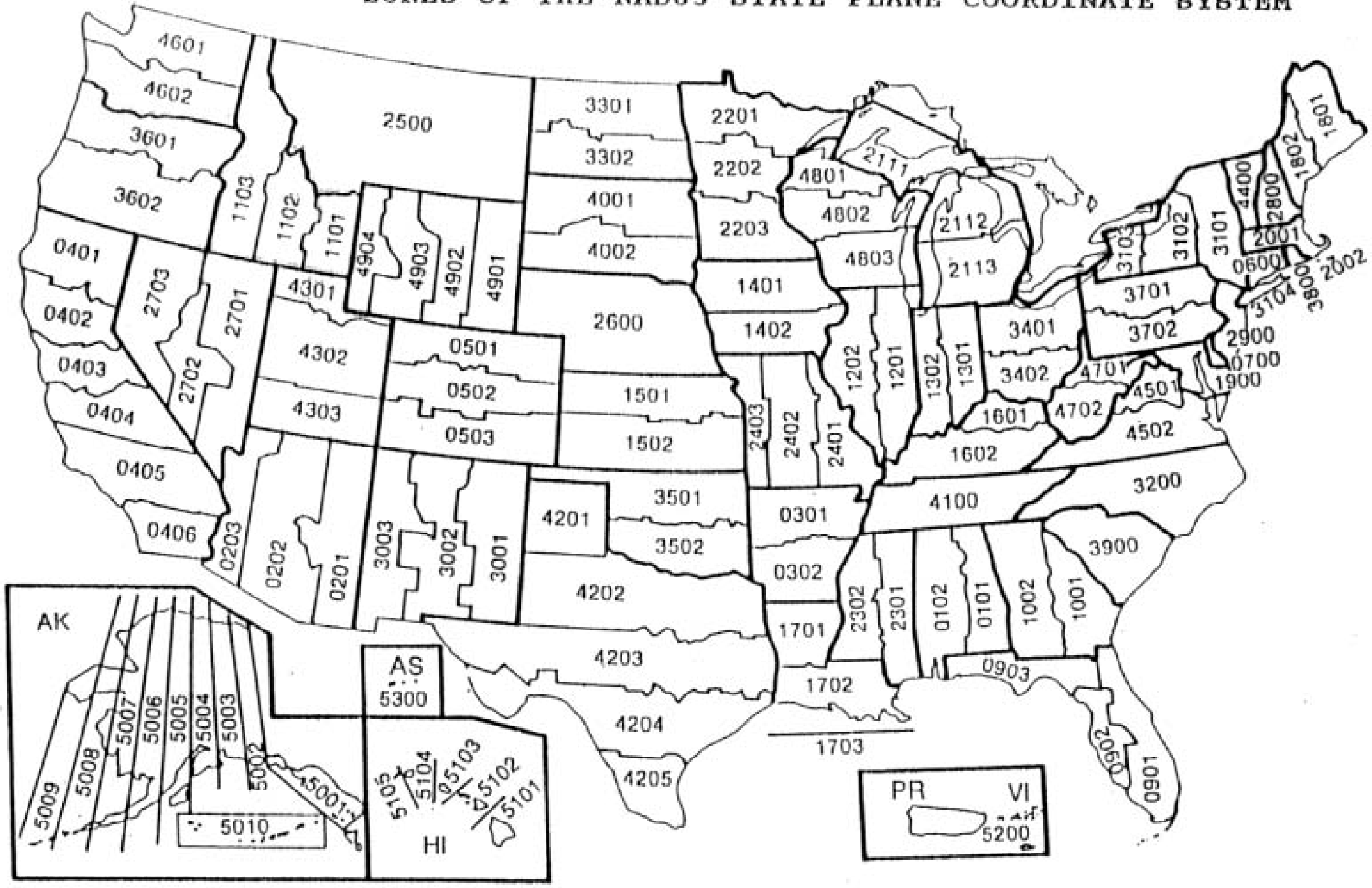
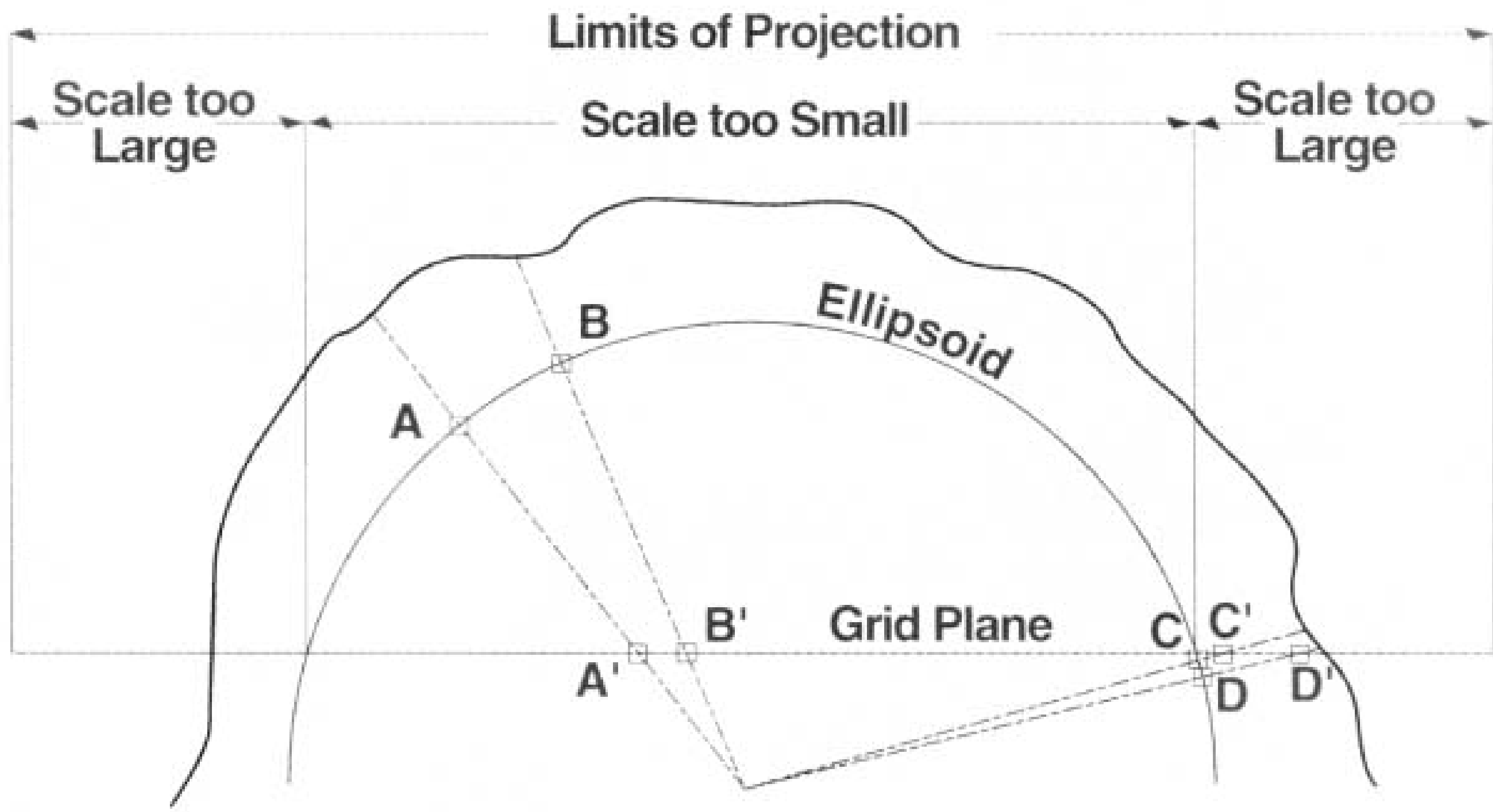


Figure C.7

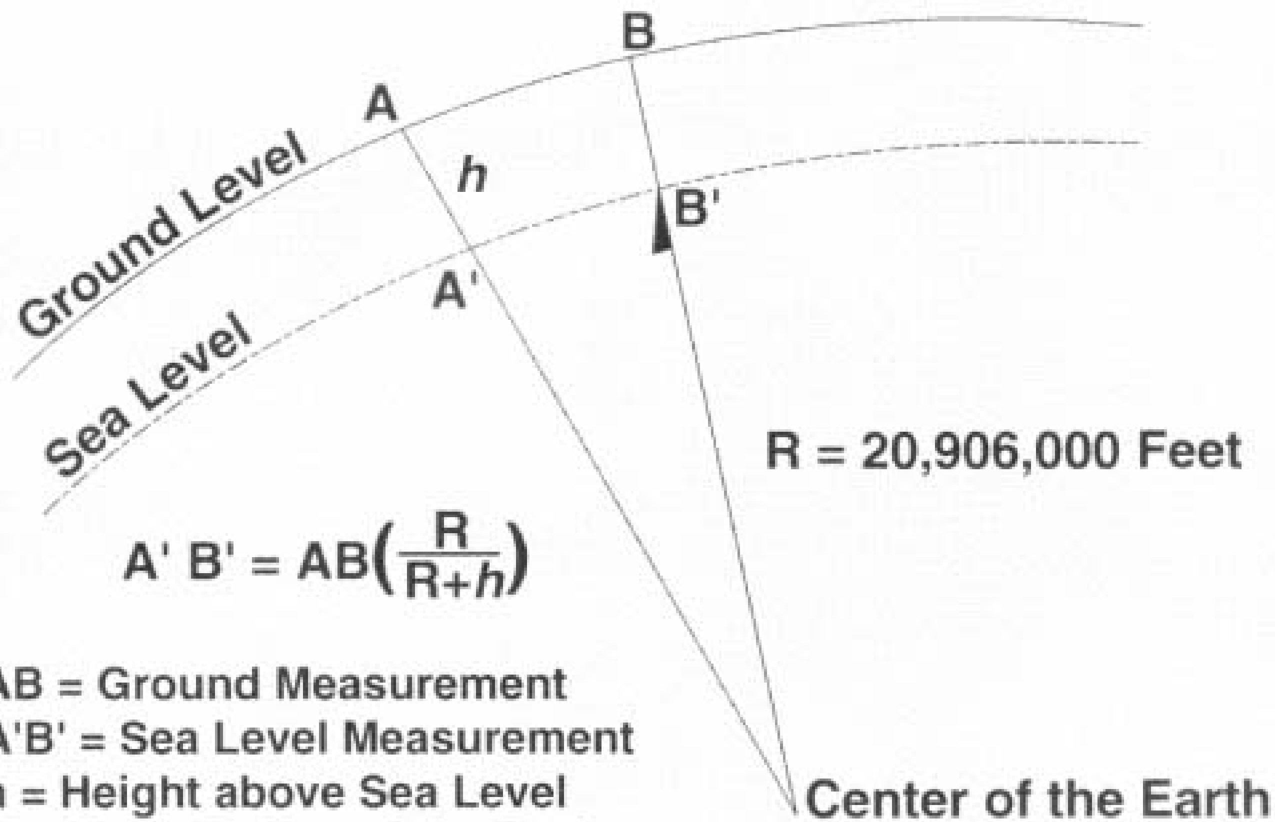
Distance Reduction Geodetic to Grid



Grid Distance A' to B' is smaller than Geodedic distance A to B
Grid Distance C' to D' is larger than Geodedic distance C to D

Figure C.8

Reduction to Sea Level

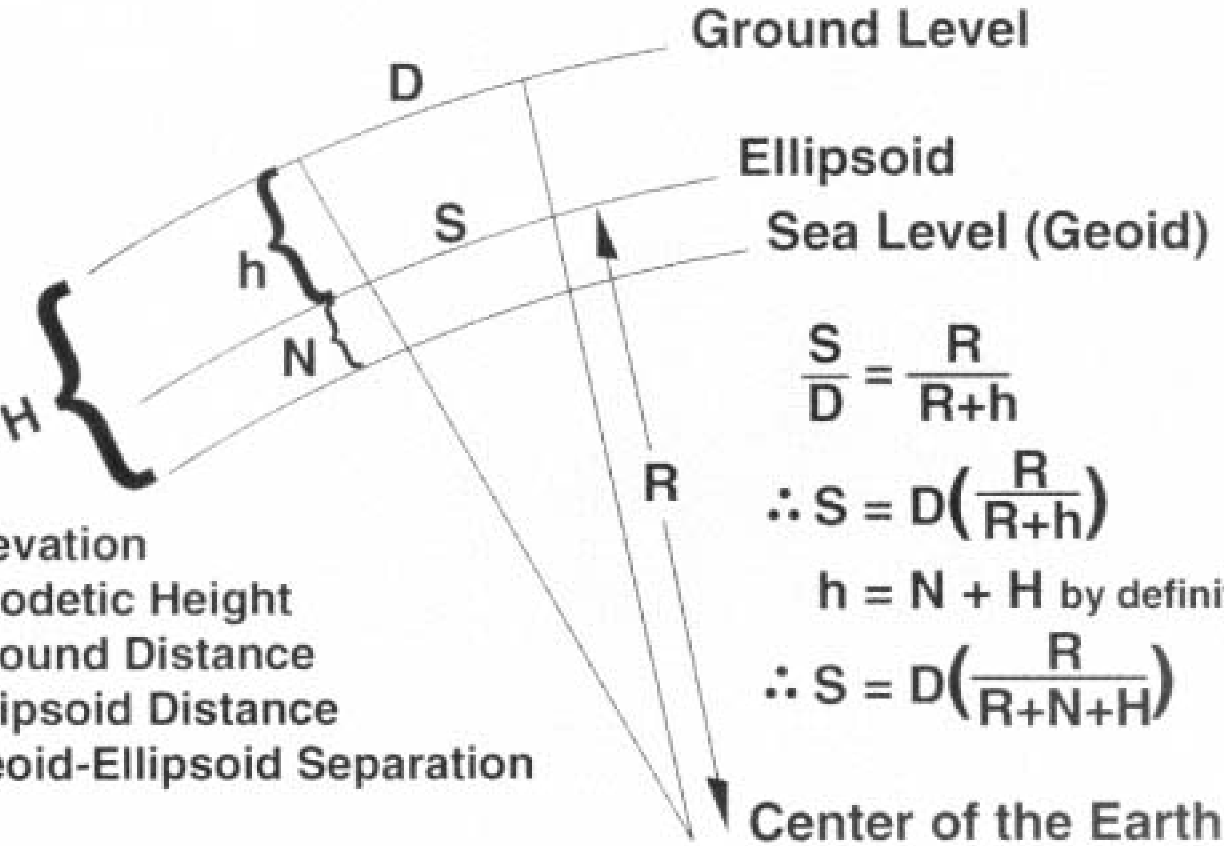


$$A'B' = AB\left(\frac{R}{R+h}\right)$$

AB = Ground Measurement
A'B' = Sea Level Measurement
h = Height above Sea Level
R = Mean Radius of the Earth

Figure C.9

Reduction to the Ellipsoid



$$\frac{S}{D} = \frac{R}{R+h}$$

$$\therefore S = D\left(\frac{R}{R+h}\right)$$

$h = N + H$ by definition

$$\therefore S = D\left(\frac{R}{R+N+H}\right)$$

H = Elevation
h = Geodetic Height
D = Ground Distance
S = Ellipsoid Distance
N = Geoid-Ellipsoid Separation

Figure C.10

TRANSVERSE MERCATOR PROJECTION

ILLINOIS

EAST ZONE

x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio	x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-108.6	0.9999750	175,000	+43.5	1.0000100
5,000	-108.5	0.9999750	180,000	+52.3	1.0000120
10,000	-108.1	0.9999751	185,000	+61.3	1.0000141
15,000	-107.5	0.9999752	190,000	+70.6	1.0000163
20,000	-106.6	0.9999755	195,000	+80.2	1.0000185
25,000	-105.5	0.9999757	200,000	+90.0	1.0000207
30,000	-104.1	0.9999760	205,000	+100.1	1.0000230
35,000	-102.5	0.9999764	210,000	+110.4	1.0000254
40,000	-100.7	0.9999768	215,000	+120.9	1.0000278
45,000	-98.5	0.9999773	220,000	+131.7	1.0000303
50,000	-96.2	0.9999778	225,000	+142.8	1.0000329
55,000	-93.6	0.9999784	230,000	+154.1	1.0000355
60,000	-90.7	0.9999791	235,000	+165.6	1.0000381
65,000	-87.6	0.9999798	240,000	+177.4	1.0000408
70,000	-84.3	0.9999806	245,000	+189.4	1.0000436
75,000	-80.7	0.9999814	250,000	+201.7	1.0000464
80,000	-76.8	0.9999823	255,000	+214.3	1.0000493
85,000	-72.7	0.9999833	260,000	+227.0	1.0000523
90,000	-68.4	0.9999843	265,000	+240.1	1.0000553
95,000	-63.8	0.9999853	270,000	+253.4	1.0000583
100,000	-58.9	0.9999864	275,000	+266.9	1.0000615
105,000	-53.9	0.9999876	280,000	+280.7	1.0000646
110,000	-48.5	0.9999888	285,000	+294.7	1.0000679
115,000	-42.9	0.9999901	290,000	+309.0	1.0000711
120,000	-37.1	0.9999915	295,000	+323.5	1.0000745
125,000	-31.0	0.9999929	300,000	+338.3	1.0000779
130,000	-24.7	0.9999943	305,000	+353.3	1.0000814
135,000	-18.1	0.9999958	310,000	+368.5	1.0000849
140,000	-11.3	0.9999974	315,000	+384.1	1.0000884
145,000	-4.2	0.9999990	320,000	+399.8	1.0000921
150,000	+3.1	1.0000007	325,000	+415.8	1.0000957
155,000	+10.7	1.0000025	330,000	+432.1	1.0000995
160,000	+18.5	1.0000043	335,000	+448.6	1.0001033
165,000	+26.6	1.0000061	340,000	+465.4	1.0001072
170,000	+34.9	1.0000080	345,000	+482.4	1.0001111
			350,000	+499.6	1.0001150

Figure C.11

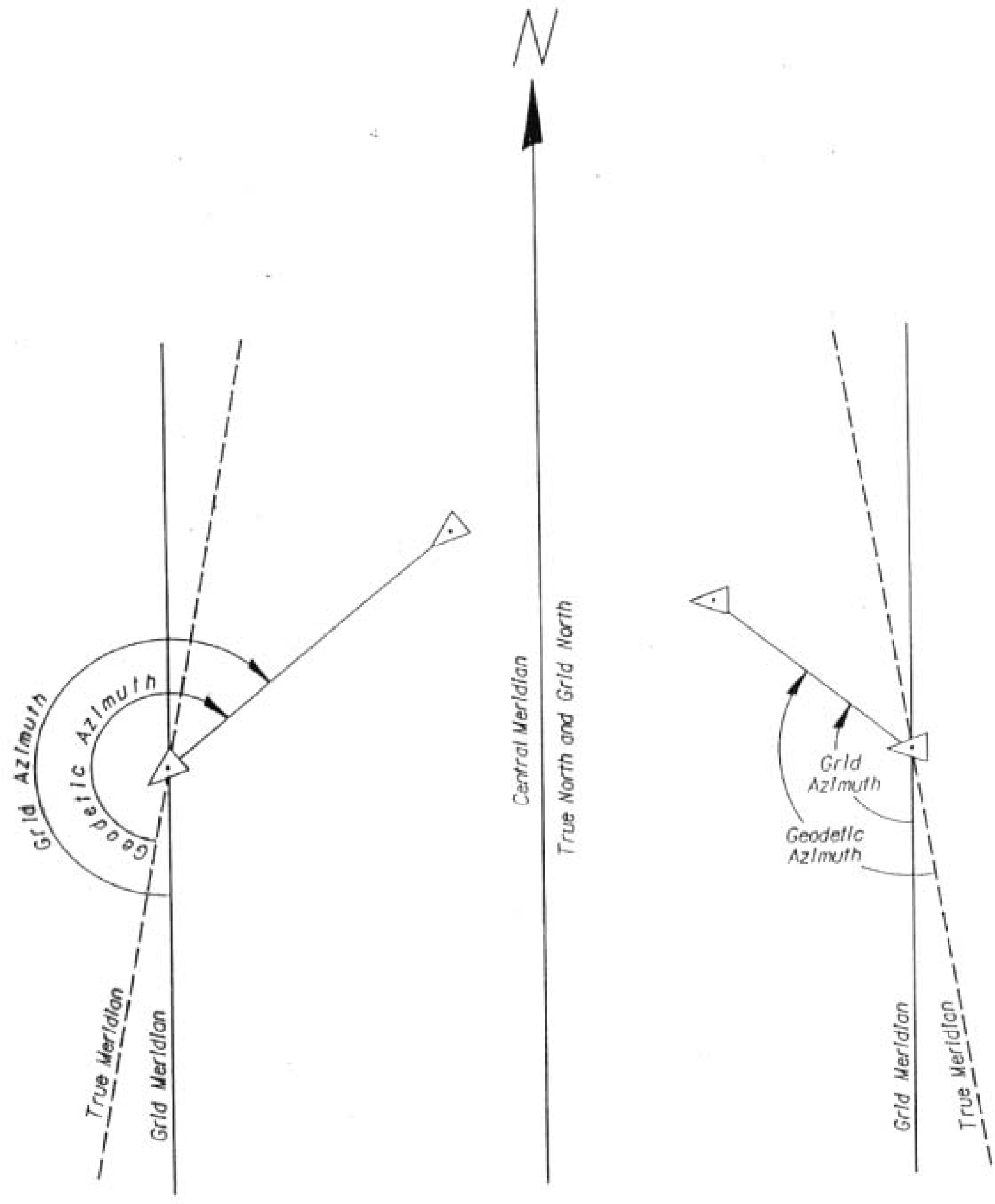
TRANSVERSE MERCATOR PROJECTION

ILLINOIS

WEST ZONE

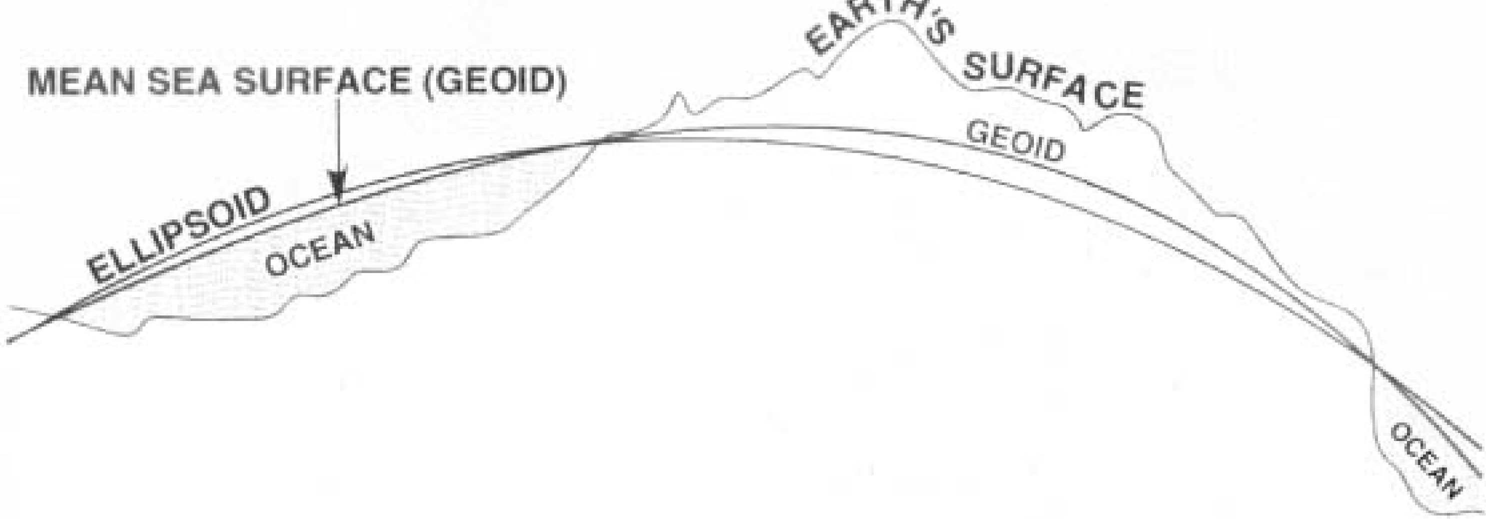
x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio	x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-255.5	0.9999412	175,000	-103.4	0.9999762
5,000	-255.4	0.9999412	180,000	-94.6	0.9999782
10,000	-255.0	0.9999413	185,000	-85.6	0.9999803
15,000	-254.4	0.9999414	190,000	-76.3	0.9999824
20,000	-253.5	0.9999416	195,000	-66.7	0.9999846
25,000	-252.4	0.9999419	200,000	-56.9	0.9999869
30,000	-251.0	0.9999422	205,000	-46.8	0.9999892
35,000	-249.4	0.9999426	210,000	-36.5	0.9999916
40,000	-247.5	0.9999430	215,000	-26.0	0.9999940
45,000	-245.4	0.9999435	220,000	-15.2	0.9999965
50,000	-243.1	0.9999440	225,000	-4.1	0.9999991
55,000	-240.5	0.9999446	230,000	+7.2	1.0000017
60,000	-237.6	0.9999453	235,000	+18.7	1.0000043
65,000	-234.5	0.9999460	240,000	+30.5	1.0000070
70,000	-231.2	0.9999468	245,000	+42.5	1.0000098
75,000	-227.6	0.9999476	250,000	+54.8	1.0000126
80,000	-223.7	0.9999485	255,000	+67.4	1.0000155
85,000	-219.6	0.9999494	260,000	+80.1	1.0000184
90,000	-215.3	0.9999504	265,000	+93.2	1.0000215
95,000	-210.7	0.9999515	270,000	+106.5	1.0000245
100,000	-205.8	0.9999526	275,000	+120.0	1.0000276
105,000	-200.7	0.9999538	280,000	+133.8	1.0000308
110,000	-195.4	0.9999550	285,000	+147.8	1.0000340
115,000	-189.8	0.9999563	290,000	+162.1	1.0000373
120,000	-184.0	0.9999576	295,000	+176.6	1.0000407
125,000	-177.9	0.9999590	300,000	+191.4	1.0000441
130,000	-171.6	0.9999605	305,000	+206.4	1.0000475
135,000	-165.0	0.9999620	310,000	+221.6	1.0000510
140,000	-158.2	0.9999636	315,000	+237.2	1.0000546
145,000	-151.1	0.9999652	320,000	+252.9	1.0000582
150,000	-143.8	0.9999669	325,000	+268.9	1.0000619
155,000	-136.2	0.9999686	330,000	+285.2	1.0000657
160,000	-128.4	0.9999704	335,000	+301.7	1.0000695
165,000	-120.3	0.9999723	340,000	+318.5	1.0000733
170,000	-112.0	0.9999742	345,000	+335.5	1.0000772

Figure C.12



GEODETIC AND GRID AZIMUTH

Figure C.13



Geoid - Ellipsoid Surface Relationships

Figure C.14